

Introduction to Deductive Logic

- An argument = an attempt to establish a thesis, or a proposition, or a sentence, to prove that this sentence is true, on the basis of certain evidence.
- We call sentences whose truth is assumed from the very beginning *premises* of an argument and we call a thesis we want to establish or prove a *conclusion* of an argument.

Premise 1

Premise 2

.....

Conclusion

An argument is a sequence of sentences (or propositions) consisting of premises and a conclusion.

Book version:

An argument is a set of two or more sentences, one of which is designated as the conclusion and the others as the premises.

- Example:

I have a headache today.

Paul McCartney is the president of UGA.

Beyoncé is the head of Food and Drug Administration.

God exists.

- “Good” vs. “bad” arguments. To be good, an argument must be *valid*.

If Mary is in Paris, she is in France.

Mary is in Paris.

Therefore, Mary is in France.

An argument is *logically valid* if and only if (= iff) it is not possible for all of its premises to be true and its conclusion false. An argument is *logically invalid* if and only if it is not logically valid.

- Validity = Truth-preservation (preservation of truth-value)
- Truth-values: **T** and **F**

- Validity is a matter of *logical form* or *structure*, not content, of an argument. It does not concern what an argument speaks about but rather the *order* in which propositions and logical words, such as ‘if’, ‘then’, ‘and’, ‘not’, ‘or’, ‘all’, ‘some’, etc., occur in it.

If Mary is in Paris, she is in France.

Mary is in Paris.

Therefore, Mary is in France.

If **p**, then **q**

p

(*Modus Ponens*)

q

If Mary Q. Public works hard in this class, then she will succeed in it.

Mary Q. Public works hard in this class.

She will succeed.

- Logic as the study of valid argument forms

Aristotelian (Syllogistic) Logic

- All A's are B's.
- No A's are B's.
- Some A's are B's.
- Some A's are not B's.

All women are humans.
All humans are mortal.

All women are mortal.

All A's are B's.
All B's are C's.

All A's are C's,

All rectangles are parallelograms.
All parallelograms have two pairs of parallel sides.

All rectangles have two pairs of parallel sides.

Some A's are B's.
All B's are C's.

All A's are C's

Some lawyers are yuppies.
All yuppies are corrupted.

All lawyers are corrupted.

➤ Problem with syllogistic logic: weak expressive power...

- “This desk is brown”
- “Seven is greater than five.” →
→ “All things that are seven are greater than five” (? :)

Logical Validity and Soundness

An argument is *logically valid* iff it is not possible for all of its premises to be true and its conclusion false. An argument is *logically invalid* iff it is not logically valid.

- To be “good,” an argument must be valid, that is, must have one of valid argument forms (catalogued by logicians).
 - Is it sufficient?

Every horse is an animal. (Every A is B)

Every animal is a movie star. (Every B is C)

Every horse is a movie star. (Every A is C)

- Validity is not enough. To be good, an argument must not only be valid, but *its premises must be in fact true*.

An argument is *logically sound* iff (1) it is logically valid and (2) all of its premises are true.

An argument is *logically unsound* iff it is not logically sound.

- Soundness is a matter of both *form* (1) and *content* (2) of an argument.

- A sound argument really *establishes* the truth of its conclusion. It *takes* one from true premises to a true conclusion by means of a valid argument form. *Both* conditions (i.e. (1) and (2)) are *necessary* for soundness.
- Validity is a necessary condition for soundness. But it is not sufficient.
- A valid argument can fail to be sound if one of its premises is false.
- Furthermore, an argument with true premises and true conclusion can also fail to be sound – if it's not valid.
Example:

Some Republicans served as Presidents.
GWB is a Republican.

GWB is a golf player.

- ❖ A logically *invalid* argument may have *any combination of truths and falsehoods as premises and conclusion*:
- All true premises and a true conclusion (as above)
- All true premises and a false conclusion
- One or more false premises and a true conclusion
- One or more false premises and a false conclusion

Van Cliburn is a great pianist. **T**
Glen Gould is a great pianist. **T**

Yuri Balashov is a great pianist. **F**

Yuri Balashov is a great pianist. **F**
Van Cliburn is a great pianist. **T**

Glen Gould is a great pianist. **T**

Van Cliburn is a great pianist. **T**
Yuri Balashov is a great pianist. **F**

David Letterman is a great pianist. **F**

- The *only* time one can determine whether an argument is logically valid, given *only* the truth-values of the premises and conclusion, is when the premises are all true and the conclusion false. If this is the case, the argument is invalid.
- In *all* other cases, knowing the actual truth-values of the premises and the conclusion is of no help in deciding whether the argument is valid or not. You have to consider *not* the actual truth-values, but whether it is *possible* for all the premises to be true and the conclusion false. If so, the argument is invalid. If not, it is valid.

Every Democrat served as the President. **F**
JFK was a Democrat. **T**

JFK served as the President. **T**

Other Core Concepts of Deductive Logic: Logical Truth, Falsity, Indeterminacy, Equivalence, Consistency, and Entailment

Again: the distinction between truth/falsity and possible truth/falsity is crucial to understanding these notions.

Logical Consistency

- The notion of a *set of sentences*: $\{\mathbf{p}, \mathbf{q}, \mathbf{r}, \dots\}$
- The “boldface notation”

A set of sentences is *logically consistent* if and only if it is possible for all the members of the set to be true, and *logically inconsistent* otherwise.

{Elton John is UK Prime Minister. California wines are terrible. It’s snowing today.}

{Ed is taller than Fred. Ed is shorter than Ted. Fred is taller than Ted.}

- Consistency versus (actual) truth

Logical Truth, Falsity and Indeterminacy

- Some sentences are true in virtue of their *form*, not content.
- E.g., any sentence having the form ‘A or not A’ is *always* true; it’s not possible for it to be false, no matter what particular sentence we substitute for A. “I am a tricycle or I am not a tricycle.”

A sentence is *logically true* if and only if it is not possible for the sentence to be false.

- A or (not A)
- If A then A (If it rains then it rains.)
- If everything is B, then Ted is B (If everyone is hardworking, then Ted is.)
- If (A if and only if B) then if (not A) then (not B)
(If Ted will go hiking if and only if Fred does, then if Ted does not go hiking, neither will Fred.)

A sentence is *logically false* if and only if it is not possible for the sentence to be true.

- A and (not A): I am a tricycle and I am not a tricycle.
- $2 + 2 = 5$?
- Information about the world?

A sentence is *logically indeterminate* if and only if it is neither logically true nor logically false.

- All *empirical* statements are logically indeterminate.

Logical Equivalence

Two sentences **p** and **q** are *logically equivalent* if and only if it is not possible for one of them to be true while the other is false.

‘It’s raining’ is equivalent to ‘It’s not the case that it’s not raining’.

- Every sentence is logically equivalent to itself.
- Equivalence versus truth/falsity:
‘Room B2 is cool’ and
‘Jere Morehead is President of UGA’
- All logically true sentences (and all logically false sentences) are logically equivalent.

Logical Entailment

A set of sentences *logically entails* a sentence if and only if it is not possible for the members of the set to be true and that sentence false.

- The set {Snow is white. Grass is green} logically entails ‘Grass is green’ and does not entail ‘Grass is not green’.
- The *unit set* of any sentence {**p**} logically entails **p**. E.g., {Snow is white} entails ‘Snow is white’.
- Any set *containing* **p** as a member entails **p**. E.g., two paragraphs above.
- A more interesting example: {If Mary is in Paris, she is in France. Mary is in Paris} entails ‘Mary is in France’.

Relationship between Logical Validity and Logical Entailment

- If an argument is logically valid, then the set of its premises logically entails the conclusion.
- But not vice versa! Some sentences are entailed by the *empty set* of sentences (\emptyset). E.g.: “I am a tricycle or I am not a tricycle.”

Special Cases of Logical Concepts

(i)

I have a headache today.

Prince William is the president of UGA.

Madonna is the head of Food and Drug Administration.

I am a tricycle or I am not a tricycle.

(ii) A logically true sentence is logically entailed by any set of sentences whatsoever, including the empty set.

- The set {I have a headache today. Prince William is the president of UGA. Madonna is the head of Food and Drug Administration} logically entails ‘I am a tricycle or I am not a tricycle’.

- The *empty set* of sentences (\emptyset) entails ‘I am a tricycle or I am not a tricycle’.

(iii) An argument whose premises form a logically inconsistent set is logically valid.

Ed is taller than Fred.

Ed is shorter than Ted.

Fred is taller than Ted.

God exists.

Ed is taller than Fred.
Ed is shorter than Ted.
Fred is taller than Ted.

God does not exist.

(iiia) A logically inconsistent set of sentences entails any sentence whatsoever.

- {Ed is taller than Fred. Ed is shorter than Ted. Fred is taller than Ted} entails ‘God exists’.
- {Ed is taller than Fred. Ed is shorter than Ted. Fred is taller than Ted} entails ‘God does not exist’.

(iv and v) All logically true sentences (and all logically false sentences) are logically equivalent.

Practice: Logical Consistency

{The United States does not support dictatorships.
In the 1980s the United States supported Iraq.
Iraq was a dictatorship throughout the 1980s. }

{Everyone who likes film classics likes *Casablanca*.
Everyone who likes Humphrey Bogart likes *Casablanca*.
Sarah likes *Casablanca* but she doesn't like most film
classics and she doesn't like Humphrey Bogart. }

{Everyone who likes film classics likes *Casablanca*.
Everyone who likes *Casablanca* likes Humphrey Bogart.
Sarah likes *Casablanca* but she doesn't like most film
classics and she doesn't like Humphrey Bogart. }

Logical Truth, Falsity, and Indeterminacy

- Bob knows everyone in the class, which includes Beth, whom he doesn't know.
 - {Bob knows everyone in the class. Beth is in this class. Bob doesn't know Beth.}
- Bob is in Paris and in Texas.
- If a sentence is not logically true, then it is logically false. True or false?
- No two false sentences are logically equivalent. True or false?
- Any argument that includes among its premises 'Everyone is corrupted' and 'I am not corrupted' is logically valid. True or false?
- Every argument all of whose premises are logically true is logically valid. True or false?
- Suppose an argument has a premise that is logically equivalent to a logical falsehood. Must the argument be logically valid?