Symbolization and Truth-Functional Connectives in SL

- Formal vs. natural languages
- Simple sentences (of English) + sentential connectives (of English) = compound sentences (of English)
- *Binary* connectives: 'and', 'or', 'if … then …', 'although', etc.
- *Unary* connectives: 'it is not the case that', 'it is believed that', 'it is possible that', etc.
- 'BHO is President' + 'and' + 'Michelle is a lawyer' = 'BHO is President and Michelle is a lawyer'
- 'It is not the case that' + 'Michelle is a lawyer' = 'It is not the case that Michelle is a lawyer'

A sentential connective is used *truth-functionally* if and only if the truth-value of the compound sentence it generates is wholly determined by the truth-values of its component sentences (whatever those truth-values are).

- A connective used truth-functionally = a *truth-functional connective*; *a* sentence generated by a truth-functional connective = a *truth-functionally compound* sentence

- For all sentences A and B, 'A and B' is true if and only is 'A' is true and 'B' is true. Ex.: 'BHO is President and Michelle is a lawyer'.

- For any sentence A, 'It is not the case that A' is true if and only if 'A' is false, and vice versa. 'It is not the case that Michelle is a lawyer'.

- BUT consider: 'It is possible that A':

'It snows today' → 'It is possible that it snows today'
'2+2=5' → 'It is possible that 2+2=5'

❖'It is possible that' is *not* a truth-functional connective.

From natural language (e.g. English) to formal language (SL)

- Simple sentences of English → atomic sentences of SL (Roman letters):
  'BHO is President' → 'B'

- Atomic sentences of SL + sentential connectives of SL → compound sentences of SL

- Sentential connectives of SL:

'&'	(ampersand)
'∨'	(wedge)
'~'	(tilde)
'⊃'	(horseshoe)
'≡'	(triple bar)

Conjunction

English	SL
BHO is President and Michelle is a lawyer	B & M

Any sentence of the form **P** & **Q**, where **P** and **Q** are sentences of SL (we use boldface letters to talk generally about arbitrary sentences of SL), is a *conjunction*. **P** and **Q** are the *conjuncts*.

Characteristic Truth-Table for Conjunction

P	**Q**	**P & Q**
T	**T**	**T**
T	**F**	**F**
F	**T**	**F**
F	**F**	**F**

English sentence → English paraphrase → sentence of SL

Original English	**English Paraphrase**	**SL**
BHO is President and Michelle is a lawyer	BHO is President <u>and</u> Michelle is a lawyer	B & M
Jenna and Laura are shopping	Jenna is shopping <u>and</u> Laura is shopping	J & L
John likes psychology, but he hates logic	John likes psychology <u>and</u> John hates logic	L & H
Although Einstein was a great physicist, he was a poor mathematician	Einstein was a great physicist <u>and</u> Einstein was a poor mathematician	P & M
Three math courses and two science courses make a full semester load	Three math courses make a full semester load <u>and</u> two science courses make a full semester load	M & S

- Part of the meaning of the original English is lost in paraphrase

- Use your linguistic competence (and good taste) to decide if a sentence can be paraphrased as a truth-functional compound and, hence, whether the sentence should be symbolized as an atomic or a compound sentence of SL.

Disjunction

English	SL
Ted is smart or Ed is smart	$T \vee E$

Any sentence of the form $\mathbf{P} \vee \mathbf{Q}$, where **P** and **Q** are sentences of SL is a *disjunction*. **P** and **Q** are the *disjuncts*.

P	**Q**	$\mathbf{P} \vee \mathbf{Q}$
T	**T**	**T**
T	**F**	**T**
F	**T**	**T**
F	**F**	**F**

- If 'or' of the original English sentence is used truth-functionally (according to the above table), the original sentence can (informally) be called a disjunction too. (Strictly speaking, only sentences of SL of the form $\mathbf{P} \vee \mathbf{Q}$ are properly called disjunctions.)
- '$\vee$' expresses the *inclusive* meaning of the English 'or'

Original English	**English Paraphrase**	**SL**
Ted is smart or Ed is smart.	Ted is smart <u>or</u> Ed is smart.	$T \vee E$
John will get an 'A' in psychology or logic.	John will get an 'A' in psychology <u>or</u> John will get an 'A' in logic.	$P \vee L$
At least one of the job candidates, Ed and Ted, will get the job.	Ed will get the job <u>or</u> Ted will get the job.	$E \vee T$
You will get a 2.9% APR or $1,000 cash back on your new car.	you will get an 2.9% APR on your new car <u>or</u> you will get $1,000 cash back on your new car.	$A \vee C$
This plant will die unless it is watered.	This plant will die <u>or</u> this plant is watered.	$D \vee W$

Negation

English	SL
It is not the case that Fred is rich.	~ F

Any sentence of the form **~ P**, where **P** is a sentence of SL is a *negation*.

P	**~ P**
T	**F**
F	**T**

- '~ ~ A' versus 'A': 'A' is *not* the negation of '~ A'

Original English	**English Paraphrase**	**SL**
Fred isn't rich.	<u>It is not the case that</u> Fred is rich.	~ F
Not all lawyers are smart.	<u>It is not the case that</u> all lawyers are smart.	~ A
No lawyers are smart.	<u>It is not the case that</u> some lawyers are smart.	~ S
Some lawyers are not smart.	<u>It is not the case that</u> all lawyers are smart.	~ A

Combining Sentential Connectives

'It is not the case that' + 'This plant will die unless it is watered' = 'It is not the case that this plant will die unless it is watered': '~' + 'W ∨ D' = '~ (W ∨ D)'

- Note the use of parentheses: Cf: '~ W ∨ D': "This plant won't die unless it is watered." Analogy: $2 \times (3 + 4)$ versus $2 \times 3 + 4$

Original English	**English Paraphrase**	**SL**
It's not the case that Carol or Bob jogs regularly; moreover, Albert doesn't jog regularly either.	It is not the case that (Carol jogs regularly or Bob jogs regularly) and it is not the case that Albert jogs regularly	~ (C ∨ B) & ~ A
It's not the case that Albert, Bob, or Carol jogs regularly.	It is not the case that (Albert jogs regularly or Bob jogs regularly) or Carol jogs regularly.	~ ((A ∨ B) ∨ C)

Original English	**English Paraphrase**	**SL**
Neither John nor Mary likes logic.	(a) It is not the case that John likes logic and it is not the case that Mary likes logic. (b) It is not the case that (John likes logic or Mary likes logic)	(a) $\sim J \,\&\, \sim M$ (b) $\sim (J \vee M)$
You will get a 2.9% APR or $1,000 cash back on your new car.	(You will get a 2.9% APR on your new car or you will get $1,000 cash back on your new car) and it is not the case that (you will get a 2.9% APR on your new car and you will get $1,000 cash back on your new car)	$(A \vee C) \,\&\, \sim (A \,\&\, C)$

Material Conditional

- '⊃' is a very special case

Suggestion: Think of **P ⊃ Q** as being equivalent, by definition, to **~ P ∨ Q**

P	Q	~ P ∨ Q	P ⊃ Q
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

A sentence of the form **P ⊃ Q**, where **P** and **Q** are sentences of SL is a *material conditional*. **P** is the *antecedent* and **Q** is the *consequent* of the conditional.

- The connection between '⊃' and conditional sentences of English is not straightforward: some conditional sentences of English can be (more or less) adequately expressed by **P ⊃ Q**, whereas others cannot.

Original English	**English Paraphrase**	**SL**
If Pluto is a dog, it is an animal.	If Pluto is a dog then Pluto is an animal. (It is not the case that Pluto is a dog or Pluto is an animal.)	$D \supset A$
If this piece of salt is put in water, it will dissolve.	If this piece of salt is put in water, then this piece of salt will dissolve.	$W \supset D$?
If this piece of salt is put in water, it will not dissolve.	If this piece of salt is put in water, then it is not the case that this piece of salt will dissolve.	$W \supset \sim D$?

- Suppose this piece of salt is *not* put in water. Then 'W' is false. This makes *both* '$W \supset D$' and '$W \supset \sim D$' *true*. But we don't want to count *both original English sentences* as being true in this case (i.e., when salt is *not* put in water). Why? Because the first seems to express a genuine law of nature (hence, a true statement), while the second doesn't.

Original English	**English Paraphrase**	**SL**
Sam will succeed in this course provided he works hard.	If Sam works hard, then Sam will succeed in this course.	$H \supset S$
Mary will come to the appointment unless she has an urgent meeting.	(a) If it is not the case that Mary has an urgent meeting then Mary will come to the appointment. (b) Mary will come to the appointment or Mary has an urgent meeting.	(a) $\sim M \supset A$ (b) $M \vee A$
Carol is a marathon runner only if she jogs regularly.	If Carol is a marathon runner then Carol jogs regularly.	$R \supset J$
Carol is a marathon runner if she jogs regularly.	If Carol jogs regularly then she is a marathon runner.	$J \supset R$

Material Biconditional

- '≡' ('if and only if') has the force of '⊃' going *both ways* (i.e., '→' and '←').

Suppose:

(If Carol is a marathon runner then Carol jogs regularly) and (if Carol jogs regularly then she is a marathon runner): (R ⊃ J) & (J ⊃ R)

A neater way to put it is:

Carol is a marathon runner if and only if Carol jogs regularly: R ≡ J

The force of '≡' can also be adequately expressed by:

(Carol is a marathon runner and Carol jogs regularly) or (it is not the case that Carol is a marathon runner and it is not the case that Carol jogs regularly): (R & J) ∨ (~ J & ~R)

P	**Q**	**(P ⊃ Q)&(Q ⊃ P)**	**(P & Q) ∨ (~ P&~Q)**	**P ≡ Q**
T	**T**	**T**	**T**	**T**
T	**F**	**F**	**F**	**F**
F	**T**	**F**	**F**	**F**
F	**F**	**T**	**T**	**T**

Common Mistakes in Symbolizing Material Conditionals and Biconditionals

- Confusing 'if' and 'if and only if'; 'if' requires '$\supset$', while 'if and only if' requires '$\equiv$'. They have **different** characteristic truth-tables!
  - If Williams loses then Sharapova will be delighted. → $W \supset S$
  - Sharapova will be delighted if Williams loses. → $W \supset S$
  - Sharapova will be delighted if and only if Williams loses. → $S \equiv W$

- Confusing 'if' and 'only if':
  - Ed will come <u>if</u> Fred cooks a dinner. → $F \supset E$
  - Ed will come <u>only if</u> Fred cooks a dinner. → $E \supset F$

- Getting confused in symbolizing 'unless'.
  - Advice: *Always* symbolize 'unless' as '$\vee$'

Complex Symbolizations

Guidelines:

- Identify simple sentences correctly
- Be careful about grouping simple sentences (i.e., those that cannot be broken down further in a truth-functional paraphrase) in a compound sentence. A simple sentence may be fairly long:
  - Ex.: 'Ted believes that Fred will get an A in the course or both Ed and Fred will get a B.' It would be **wrong** to break it down as: Ted believes that Fred will get an A in the course <u>or</u> (Ted believes that Ed will get a B' in the course <u>and</u> Ted believes that Fred will get a 'B' in the course)': 'A $\vee$ (B_1 & B_2)'. You have to abbreviate the entire original sentence as a simple sentence: e.g., 'T'.
- Use parentheses to eliminate ambiguities:
  - Ex.: '~ (W $\vee$ D)' versus '~ W $\vee$ D'
- Where an English passage contains multiple wordings or tenses of the same claim, use, wherever appropriate, *one* wording or tense all throughout in constructing a truth-functional paraphrase:
  - Ex.: 'Ted will not come to the meeting' is equavalent to 'Ted will miss the meeting'
- Substitute actual proper names for pronouns in paraphrases:
  - Ex.: 'If Pluto is a dog, it is an animal' → '<u>If</u> Pluto is a dog <u>then</u> Pluto is an animal'

F: The French team will win at least one gold medal.
G: The German team will win at least one gold medal.
D: The Danish team will win at least one gold medal.
P: The French team is plagued with injuries
S: The star German runner is disqualified
R: It rains during most of the competition.

(1) At most one of the French, German, or Danish teams will win a gold medal.

It is not the case that (the French team will win at least one gold medal or the German team will win at least one gold medal) or [it is not the case that (the French team will win at least one gold medal or the Danish team will win at least one gold medal) or it is not the case that (the German team will win at least one gold medal or the Danish team will win at least one gold medal)].

$$\sim (F \vee G) \vee [\sim (F \vee D) \vee \sim (G \vee D)]$$

(2) They will all win gold medals.

The French team will win at least one gold medal and (the German team will win at least one gold medal and the Danish team will win at least one gold medal).

$$F \,\&\, (G \,\&\, D)$$

F: The French team will win at least one gold medal.
G: The German team will win at least one gold medal.
D: The Danish team will win at least one gold medal.
P: The French team is plagued with injuries.
S: The star German runner is disqualified.
R: It rains during most of the competition.

(3) The French will win a gold medal only if they are not plagued with injuries, in which case they won't win.

(If the French team will win at least one gold medal then it is not the case that the French team is plagued with injuries) and (if the French team is plagued with injuries then it is not the case that the French team will win at least one gold medal).

$$(F \supset \sim P) \ \& \ (P \supset \sim F)$$

(4) Provided it doesn't rain during most of the competition and their star runner isn't disqualified, the Germans will win a gold medal if either of the other teams does.

If (it is not the case that it rains during most of the competition and it is not the case that the star German runner is disqualified) then [if (the French team will win at least one gold medal or the Danish team will win at least one gold medal) then the German team will win at least one gold medal].

$$(\sim R \ \& \sim S) \supset [(F \vee D) \supset G]$$

F: The French team will win at least one gold medal.
G: The German team will win at least one gold medal.
D: The Danish team will win at least one gold medal.
P: The French team is plagued with injuries.
S: The star German runner is disqualified.
R: It rains during most of the competition.

(5) The Germans will win a gold medal only if it doesn't rain during most of the competition and their star runner is not disqualified.

If the German team will win a gold medal then it is not the case that (it rains during most of the competition or the star German runner is disqualified).

$$G \supset \sim (R \vee S)$$

(6) The Danes will win a gold medal unless it rains during most of the competition, in which case they won't but the other two teams will win gold medals.

(The Danish team will win at least one gold medal or it rains during most of the competition) and (if it rains during most of the competition then [it is not the case that the Danish team will win at least one gold medal and (the German team will win at least one gold medal and the French team will win at least one gold medal)]).

$$(D \vee R) \,\&\, (R \supset [\sim D \,\&\, (G \,\&\, F)])$$

Symbolizing Arguments

Assuming Betty is the judge, Peter won't get a suspended sentence. The trial will be long unless the district attorney is brief, but the district attorney is not brief. Fred is the defense lawyer. However, if Fred is the defense lawyer, Peter will be found guilty; and if Peter will be found guilty, he will be given a sentence. Consequently, after a long trial Peter will be given a sentence that won't be suspended by the judge.

B: Betty is the judge.
P: Peter will be given a suspended sentence.
T: The trial will be long.
D: The district attorney is brief.
F: Fred is the defense lawyer.
G: Peter will be found guilty.
S: Peter will be given a sentence.

If Betty is the judge then it is not the case that Peter will be given a suspended sentence.	$B \supset \sim P$
(The trial will be long or the district attorney is brief) and it is not the case that the district attorney is brief.	$(T \vee D) \& \sim D$
Fred is the defense attorney.	F
(If Fred is the defense lawyer then Peter will be found guilty) and (if Peter will be found guilty then Peter will be given a sentence).	$(F \supset G) \& (G \supset S)$
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The trial will be long and (Peter will be given a sentence and it is not the case that Peter will be given a suspended sentence)	$T \& (S \& \sim P)$

The Syntax of SL

Syntax: the study of the expressions of a language (e.g., a formal language of logic), the relations among them, and the rules for constructing more complex expressions out of less complex ones, without regard to questions of interpretation.

Use versus Mention. Object Language and Metalanguage

- We *use* words to talk about something else: e.g., things that they name or denote.
- We *mention* words (and whole expressions), by putting them in ' ' or displaying them in some conspicuous way, to talk about words (expressions) themselves.

- Ted is a student.
- 'Ted' has three letters.

- Snow is white
- 'Snow is white' is a true sentence

…………………………………………………

Snow is white

is a true sentence.

…………………………………………………..

- We can talk about a language (such as the formal language of SL) *by means of* another language (e.g., English).
- The language we talk *about* (i.e., study, describe, analyze) is the object language.
- The language *in which* we talk about (study, describe) the object language is the *metalanguage*.
- Expressions of the object language are *mentioned*, and *not used*, in the metalanguage.

Use:

A ⊃ B
A

B

Mention:

'A ⊃ B' is a material conditional.

Metavariables

- Metavariables (boldfaced letters '**P**', '**Q**', '**R**', and '**S**') are used to talk about certain sorts of expressions of SL (e.g., sentences) *generally*.

- 'A ⊃ B' is a material conditional.
- For any two sentences of SL, **P** and **Q**, **P** ⊃ **Q** is a material conditional.

- Similarly, metavariables '**p**', '**q**', '**r**', and '**s**') are used to talk about certain sorts of expressions of English (e.g., sentences) *generally*. (E.g., page 54.)

The Language of SL

- What are the basic expressions of SL?
- How can we build other expressions from the basic ones?

Vocabulary of SL

Sentence Letters	Sentential Connectives	Punctuation Marks
'A', 'B', … , 'A_1', 'B_1', …	'~', '&', '∨', '⊃', '≡'	'(', ')' ('[', ']')

Recursive Definition of 'Sentence of SL'

1. Every sentence letter of SL is a sentence of SL.
2. If **P** is a sentence of SL, then ~**P** is a sentence of SL.
3. If **P** and **Q** are sentences of SL, then (**P** & **Q**) is a sentence of SL.
4. If **P** and **Q** are sentences of SL, then (**P** ∨ **Q**) is a sentence of SL.
5. If **P** and **Q** are sentences of SL, then (**P** ⊃ **Q**) is a sentence of SL.
6. If **P** and **Q** are sentences of SL, then (**P** ≡ **Q**) is a sentence of SL.
7. Nothing is a sentence unless it can be formed by repeated application of clauses 1–6. ["That's all, folks!"]

Which of the following are sentences of SL?

& H
M ~ N
P ∨ **Q**
(U & C & ~ L)
[(G ∨ E) ⊃ (~ H & (K ∨ B))]

- Convention: The outermost parentheses of a sentence may be dropped if a sentence stands alone: '(U & (C ∨ ~ L))' → 'U & (C ∨ ~ L)'

Main Connective, Immediate Components, Components, Atomic Components of Sentences of SL

1. An atomic sentence **P** contains no connectives, does not have a main connective, and has no immediate components.
2. If **P** has the form ~ **Q**, where **Q** is a sentence, then the main connective of **P** is the ' ~ ' before **Q**, and **Q** is the immediate sentential component of **P**.
3. If **P** has the form **Q** & **R**, **Q** ∨ **R**, **Q** ⊃ **R**, or **Q** ≡ **R**, where **Q** and **R** are sentences, then the main connective of **P** is the connective that occurs between **Q** and **R**, and **Q** and **R** are the immediate sentential components of **P**.
4. The (*sentential*) *components* of a sentence include all of the following: (a) the sentence itself, (b) its immediate sentential components, and (c) the (sentential) components of its immediate components.
5. The *atomic components* of a sentence are all its (sentential) components that are atomic sentences.

- Specify the main connective and all the (sentential) components of the following sentence, indicating which are its immediate components and which are its atomic components:

$$M \supset ([\sim N \supset (B \,\&\, C)] \equiv \sim [(L \supset J) \vee X])$$

- Which of the following sentences have the form **~P ⊃Q**?

~ L ⊃ ~ J	~ [(L ⊃ J) ∨ X] ⊃ (L ⊃ J)
~ (L ⊃ ~ J)	[~ K ⊃ (L ⊃ ~ J)] ⊃ (~ J ⊃ ~ J)
~ ~ L ⊃ ~ J	~ (A ∨ B) ⊃ (~ C ≡ D)