Semantics for SL (Truth-Tables)

- Basic semantic concepts in SL: truth-functional truth / falsity / indeterminacy / consistency / entailment / validity
- The truth-value of compound sentences of SL is uniquely determined by the truth-values of its atomic components.

A *truth-value assignment* is an assignment of truth-values to the atomic sentences of SL (*all of them*, strictly speaking)

- Format and Strategies. Your knowledge of the characteristic truth-tables for the connectives must be **solid**.

↓

E	F	J	(J & [(E ∨ F) & (~ E & ~ F)]) ⊃ ~ J
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

- Full and shortened truth-tables

E F J	(J& [(E ∨ F) & (~ E & ~ F)]) ⊃ ~ J
T T T	T F T T T F F T F F T **T** F T

Logical truth / falsity: A sentence is logically true (false) just in case it is not possible for it to be false (true).

↓

Truth-functional truth / falsity: A sentence of SL is truth-functionally true (false) if and only if it is true (false) on every truth-value assignment.

Logical indeterminacy: A sentence is logically indeterminate just in case it is neither logically true nor logically false.

↓

Truth-functional indeterminacy: A sentence of SL is truth-functionally indeterminate if and only if it is neither truth-functionally true nor truth-functionally false.

- What does it take to demonstrate that a sentence is (is not) truth-functionally true, false, or indeterminate?

- Truth-functionally false:

↓

B	D	~	(D	⊃	[(B	∨	D)	⊃	D])
T	T	**F**	T	T	T	T	T	T	T
T	F	**F**	F	T	T	T	F	F	F
F	T	**F**	T	T	F	T	T	T	T
F	F	**F**	F	T	F	F	F	T	F

- Truth-functionally indeterminate:

↓

B	D	(~	B	&	~	D)	∨	~	(B	∨	D)
T	T	F	T	F	F	T	**F**	F	T	T	T
T	F	F	T	F	T	F	**F**	F	T	T	F
F	T	T	F	F	F	T	**F**	F	F	T	T
F	F	T	F	T	T	F	**T**	T	F	F	F

- Not truth-functionally true: ↓

B	D	(~	B	&	~	D)	∨	~	(B	∨	D)
T	T	F	T	F	F	T	**F**	F	T	T	T

- Not truth-functionally false: ↓

B	D	(~	B	&	~	D)	∨	~	(B	∨	D)
F	F	T	F	T	T	F	**T**	T	F	F	F

Two sentences of SL are *truth-functionally equivalent* iff there is no truth-value assignment on which they have different truth-values.

- Symbolize the following pair of sentences and determine whether they are truth-functionally equivalent:
  Although the new play at the Roxy is a flop, critics won't ignore it unless it is canceled.
  The new play at the Roxy is a flop, and if it is canceled, critics will ignore it.

N: The new play at the Roxy is a flop.
I: Critics will ignore the new play at the Roxy.
C: The new play at the Roxy is cancelled.

$N \,\&\, (\sim I \vee C)$ $N \,\&\, (C \supset I)$

C	I	N	N	↓ &	(~	I	∨	C)	N	↓ &	(C	⊃	I)
T	T	T	T	**T**	F	T	T	T	T	**T**	T	T	T
T	T	F	F	**F**	F	T	T	T	F	**F**	T	T	T
T	F	T	T	**T**	T	F	T	T	T	**F**	T	F	F
T	F	F	F	**F**	T	F	T	T	F	**F**	T	F	F
F	T	T	T	**F**	F	T	F	F	T	**T**	F	T	T
F	T	F	F	**F**	F	T	F	F	F	**F**	F	T	T
F	F	T	T	**T**	T	F	T	F	T	**T**	F	T	F
F	F	F	F	**F**	T	F	T	F	F	**F**	F	T	F

- $\sim C \supset \sim B$ and $B \supset C$

C	B	~	C	**⊃** ↓	~	B	B	**⊃** ↓	C
T	T	F	T	**T**	F	T	T	**T**	T
T	F	F	T	**T**	T	F	F	**T**	T
F	T	T	F	**F**	F	T	T	**F**	F
F	F	T	F	**T**	T	F	F	**T**	F

- A (pedestrian) set theory:
- {A, B ⊃ (C ⊃ B), D ∨ [~ (C & A)]} – a set of three sentences of SL (note the convention to drop ' ')
- 'A', 'B ⊃ (C ⊃ B)', and 'D ∨ [~ (C & A)]' are *members* of {A, B ⊃ (C ⊃ B), D ∨ [~ (C & A)]}
- ∅ is (the name of) the *empty set*
- The variable 'Γ' (Greek *gamma*) is used to talk *generally* about *sets of sentences* of SL (just as metavariables '**P**', '**Q**', etc. are used to talk generally about sentences of SL)
- {**P**} is the *unit set* of **P**
- The *union* of two sets Γ_1 and Γ_2, $\Gamma_1 \cup \Gamma_2$, is a set containing all and only members of Γ_1 and Γ_2.

A set of sentences of SL is *truth-functionally consistent* iff there is at least one truth-value assignment on which all the members of the set are true (and *truth-functionally inconsistent* otherwise).

- {A} – ?
- {A, ~ A} – ?
- {H ≡ (~ H ⊃ H)} – ?
- {~ (~ C ∨ ~ B) & A, A ≡ ~ C} – ?

A	B	C	~	(~	C	∨	~	B)	& ↓	A	A	≡ ↓	~	C
T	T	T	T	F	T	F	F	T	**T**	T	T	**F**	F	T
T	T	F	F	T	F	T	F	T	**F**	T	T	**T**	T	F
T	F	T	F	F	T	T	T	F	**F**	T	T	**F**	F	T
T	F	F	F	T	F	T	T	F	**F**	T	T	**T**	T	F
F	T	T	T	F	T	F	F	T	**F**	F	F	**T**	F	T
F	T	F	F	T	F	T	F	T	**F**	F	F	**F**	T	F
F	F	T	F	F	T	T	T	F	**F**	F	F	**T**	F	T
F	F	F	F	T	F	T	T	F	**F**	F	F	**F**	T	F

- {A, A & ~ B, B ∨ C} – ?

A	B	C	A ↓	A	& ↓	~	B	B	∨ ↓	C
T	F	T	**T**	T	**T**	T	F	F	**T**	T

- Newtonian mechanics can't be right if Einsteinian mechanics is. But Einsteinian mechanics is right if and only if space is non-Euclidean. Space is non-Euclidean, or Newtonian mechanics is correct.

  N: Newtonian mechanics is right.
  E: Einsteinian mechanics is right.
  S: Space is non-Euclidean.

$$\{E \supset \sim N,\ E \equiv S,\ S \vee N\}$$

E	N	S	E	⊃ ↓	~	N	E	≡ ↓	S	S	∨ ↓	N
T	F	T	T	**T**	T	F	T	**T**	T	T	**T**	F

Some interesting results:

- {**P**} is truth-functionally inconsistent iff **~P** is truth-functionally true.
- If **P** ≡ **Q** is truth-functionally true, then {**P**, ~ **Q**} is truth-functionally *in*consistent. (The converse *doesn't* hold.)

Truth Functional Entailment and Validity

A set Γ of sentences of SL *truth-functionally entails* a sentence **P** of SL iff there is no truth-value assignment on which all the members of Γ are true and **P** is false.

$\Gamma \vDash \mathbf{P}$ $\Gamma \nvDash \mathbf{P}$

$\{\sim A, B\} \vDash B$ (note a convention to drop '   ')

$\{\sim A, B\} \nvDash A$

$\{A, A \supset C\} \vDash C$

$\varnothing \vDash \mathbf{P}$ → $\vDash \mathbf{P}$

$\vDash A \vee \sim A$

- $\{(C \supset D) \supset (D \supset E), D\} \vDash C \supset E$?

↓ ↓ ↓

C	D	E	(C	⊃	D)	⊃	(D	⊃	E)	D	C	⊃	E
T	T	T	T	T	T	**T**	T	T	T	**T**	T	**T**	T
T	T	F	T	T	T	**F**	T	F	F	**T**	T	**F**	F
T	F	T	T	F	F	**T**	F	T	T	**F**	T	**T**	T
T	F	F	T	F	F	**T**	F	T	F	**F**	T	**F**	F
F	T	T	F	T	T	**T**	T	T	T	**T**	F	**T**	T
F	T	F	F	T	T	**F**	T	F	F	**T**	F	**T**	F
F	F	T	F	T	F	**T**	F	T	T	**F**	F	**T**	T
F	F	F	F	T	F	**T**	F	T	F	**F**	F	**T**	F

An argument of SL is *truth-functionally valid* iff there is no truth-value assignment on which all the premises are true and the conclusion false (and *truth-functionally invalid* otherwise).

- An argument of SL is truth-functionally valid iff the set consisting of its premises truth-functionally entails the conclusion.

K ≡ L

L ⊃ J ?

~ J

~ K ∨ L

				↓			↓		↓				↓	
K	L	J	K	≡	L	L	⊃	J	~	J	~	K	∨	L
T	T	T	T	**T**	T	T	**T**	T	**F**	T	F	T	**T**	T
T	T	F	T	**T**	T	T	**F**	F	**T**	F	F	T	**T**	T
T	F	T	T	**F**	F	F	**T**	T	**F**	T	F	T	**F**	F
T	F	F	T	**F**	F	F	**T**	F	**T**	F	F	T	**F**	F
F	T	T	F	**F**	T	T	**T**	T	**F**	T	T	F	**T**	T
F	T	F	F	**F**	T	T	**F**	F	**T**	F	T	F	**T**	T
F	F	T	F	**T**	F	F	**T**	T	**F**	T	T	F	**T**	F
F	F	F	F	**T**	F	F	**T**	F	**T**	F	T	F	**T**	F

B & F
~ (B & G) ?

G

		↓		↓				↓
B F G	B	&	F	~	(B	&	G)	G
T T F	T	**T**	T	**T**	T	F	F	**F**

➢ Don't worry about the *corresponding material conditional* for an argument of SL!

The town hall is now a grocery store, and, unless I'm mistaken, the little red schoolhouse is a movie theater. No, I'm not mistaken. The old schoolhouse is a boutique, and the old theater is an elementary school if the little read schoolhouse is a movie theater. So the little red schoolhouse is a movie theater.

T:	The town hall is now a grocery store.	$T \& (I \vee L)$
I:	I'm mistaken.	$\sim I$
L:	The little red schoolhouse is a movie theater.	$O \& (L \supset E)$
O:	The old school bus is a boutique.	------------
E:	The old theater is an elementary school.	L

Some Theoretical Results

- A sentence **P** is truth-functionally false iff the set {**P**} is truth-functionally inconsistent.
- A sentence **P** is truth-functionally true iff the set {~ **P**} is truth-functionally inconsistent.
- A sentence **P** is truth-functionally indeterminate iff both {**P**} and {~ **P**} are truth-functionally consistent.
- Sentences **P** is **Q** are truth-functionally equivalent iff their *corresponding material biconditional* **P** ≡ **Q** is truth-functionally true.
- Sentences **P** is **Q** are truth-functionally equivalent iff the set {~ **P** ≡ **Q** }is truth-functionally inconsistent.
- $\Gamma \vDash \mathbf{P}$ iff $\Gamma \cup \{\sim \mathbf{P}\}$ is truth-functionally inconsistent.

- An inconsistent set of sentences truth-functionally entails any sentence (whatsoever).
- A truth-functionally true sentence is entailed by any set of sentences (whatsoever).
- **P** is truth-functionally true iff $\vDash \mathbf{P}$.
- $\Gamma \vDash \mathbf{P} \supset \mathbf{Q}$ iff $\Gamma \cup \{\mathbf{P}\} \vDash \mathbf{Q}$.

Some exercises:

True or false?

- If {**P**} is truth-functionally consistent, then so is {~ **P**}.
- If {**P**} is truth-functionally inconsistent, then so is {~ **P**}.
- A material conditional with a truth-functionally false consequent must itself be truth-functionally false.

Circle the main connective of each of the following sentences and underline the immediate sentential component(s).

$[(J \vee K) \supset K] \vee \sim J$

$[A \supset [B \supset (C \equiv D)]] \supset \sim A$

$[\sim K \supset ([L \equiv M] \supset [M \,\&\, (L \vee \sim K)])] \supset ([L \,\&\, (\sim K \vee K)] \equiv (K \supset [K \supset ([(L \supset M) \supset K] \supset L)]))$

How many rows are there in the full truth-table for each of the above sentences?